Certified Core-Guided MaxSAT Solving

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NordConsNet Workshop 2023

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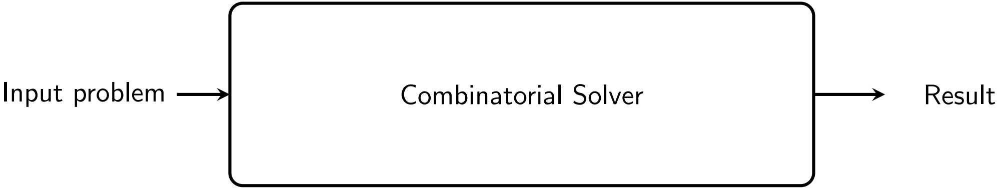
Joint work with Jeremias Berg, Bart Bogaerts, Jakob Nordström and Dieter Vandesande
to appear at CADE-29

Combinatorial Solving & Optimization



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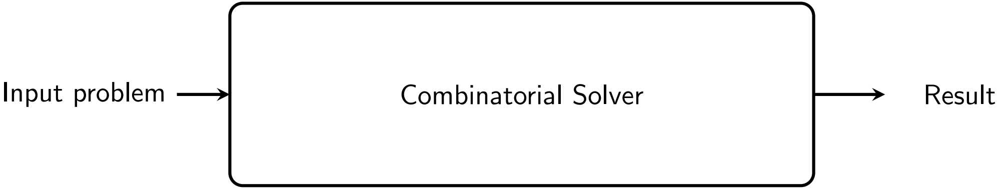

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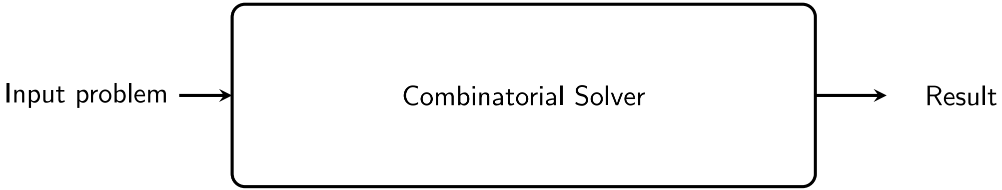

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How do we know if problem was solved correctly?

Correctness of Combinatorial Solvers

Testing:

- Can only show presence of bugs, not absence
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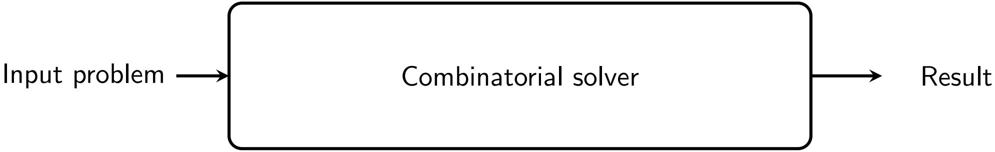
Proof logging (our approach):

- Guarantee that execution was correct
- Moderate overhead for implementing solver

Certifying Results with Proof Logging



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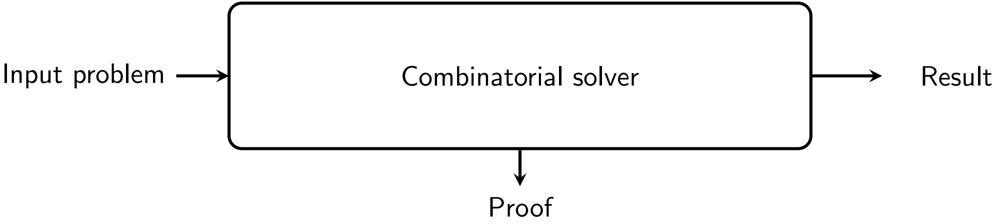

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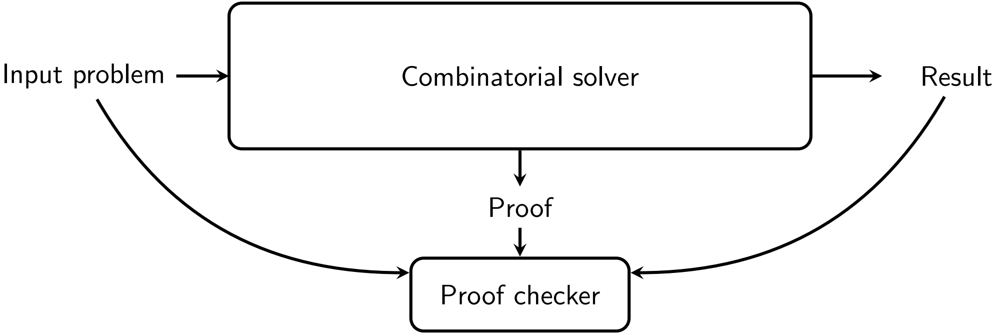

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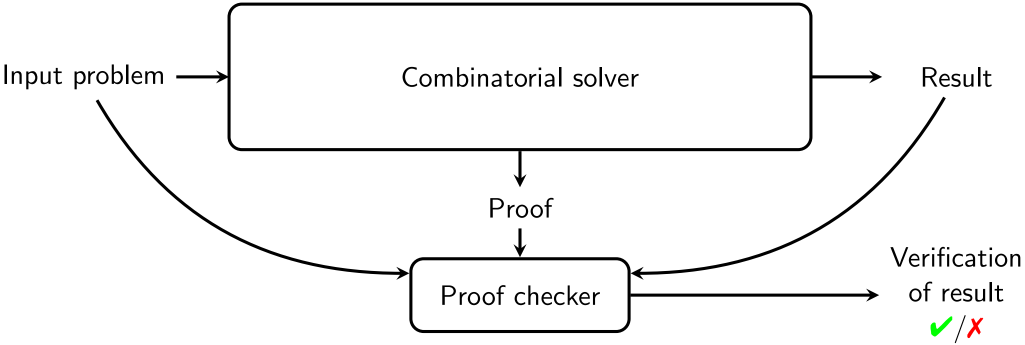

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Our Focus: Maximum Satisfiability (MaxSAT) Solving

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optimum 1

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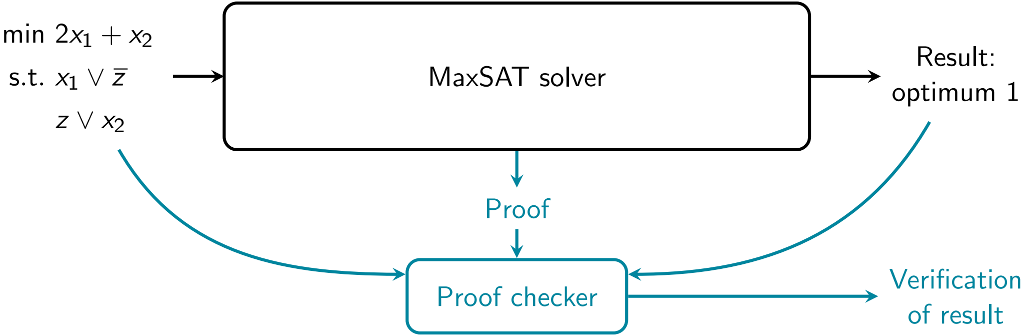
- Minimize objective subject to satisfying formula in conjunctive normal form (CNF)
- Equivalently: Maximize satisfied soft clauses subject to satisfying hard clauses
- Main approaches:
  - Solution-improving or linear SAT-UNSAT search [ES06, LP10, PRB18]
  - Implicit hitting set (IHS) search [DB13a, DB13b]
  - **Core-guided search** [FM06, NB14, ADR15, AG17]

Certified Maximum Satisfiability (MaxSAT) Solving



- This work: Certification of state-of-the-art core-guided MaxSAT solving

Rest of This Talk

1. Description of state-of-the-art core-guided MaxSAT solving
2. Our contribution: Proof logging for core-guided MaxSAT solving
3. Experimental evaluation
4. Conclusion

Basic Notation

- Boolean variable x: Domain 0 (false) and 1 (true)
- Literal ℓ: x or negation $\overline{x} = 1 - x$
- Pseudo-Boolean (PB) constraint: Integer linear inequality over literals

$$3x_1 + 2x_2 + 5\overline{x}_3 \geq 5$$

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$$3x_1 + 2x_2 + 5\overline{x}_3 = 5 \longrightarrow \begin{matrix} 3x_1 + 2x_2 + 5\overline{x}_3 \geq 5 \\ 3x_1 + 2x_2 + 5\overline{x}_3 \leq 5 \end{matrix}$$

- Clause: Disjunction of literals or at-least-one constraint

$$x_1 \vee \overline{x}_2 \vee \overline{x}_3 \iff x_1 + \overline{x}_2 + \overline{x}_3 \geq 1$$

- CNF formula can be viewed as a collection of pseudo-Boolean constraints

OLL-Style Core-Guided MaxSAT Solving [MDM14]

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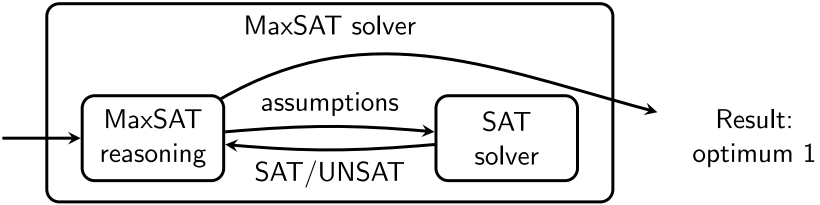

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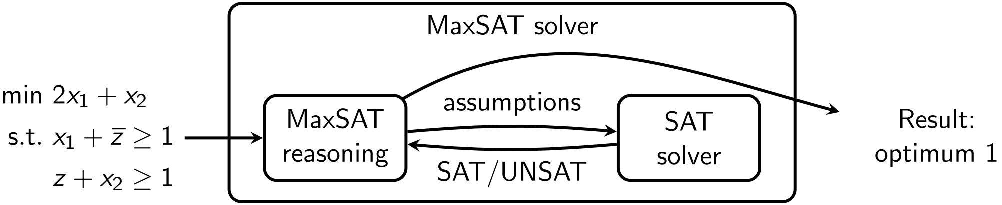

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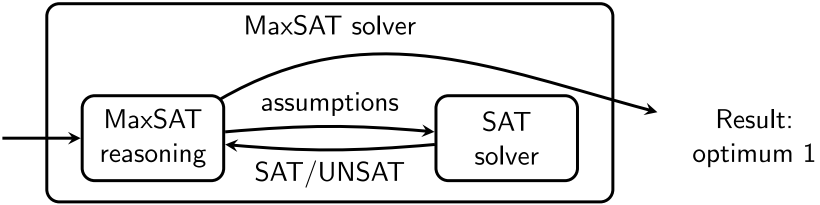

1. Try best objective value (using optimistic assumptions about the objective)
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3. Reformulate objective and goto 1.

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Rules:

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Extended Cutting Planes: Reification

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$$a \Leftrightarrow x_1 + \bar{x}_2 + 2x_3 \geq 2 \longrightarrow \begin{array}{ll} 2\bar{a} + x_1 + \bar{x}_2 + 2x_3 \geq 2 & (a \Rightarrow x_1 + \bar{x}_2 + 2x_3 \geq 2) \\ 3a + \bar{x}_1 + x_2 + 2\bar{x}_3 \geq 3 & (a \Leftarrow x_1 + \bar{x}_2 + 2x_3 \geq 2) \end{array}$$

Example: Proof Logging for Core-Guided MaxSAT Solving (1/3)

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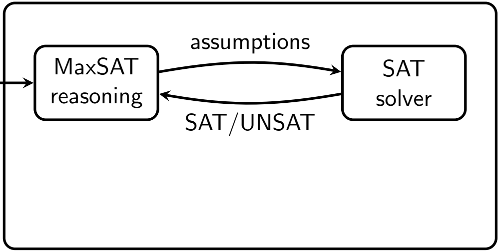

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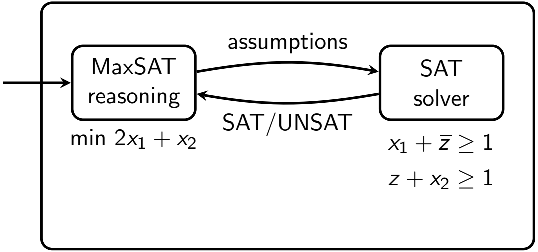

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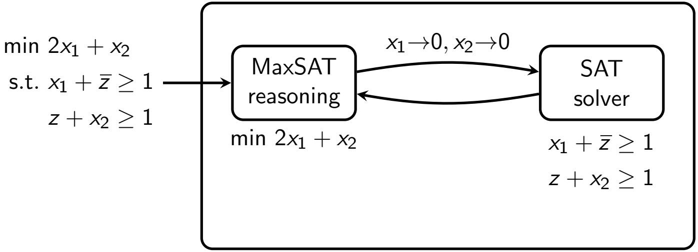

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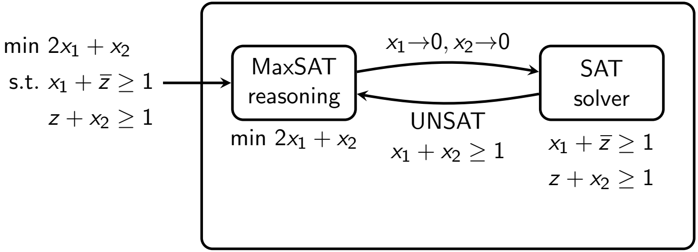

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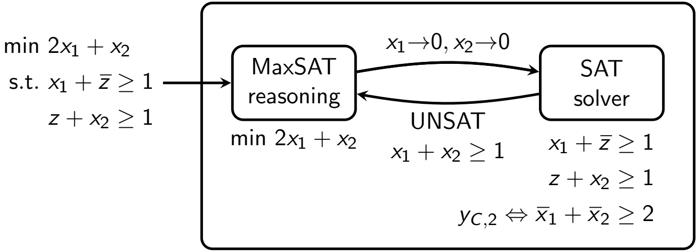

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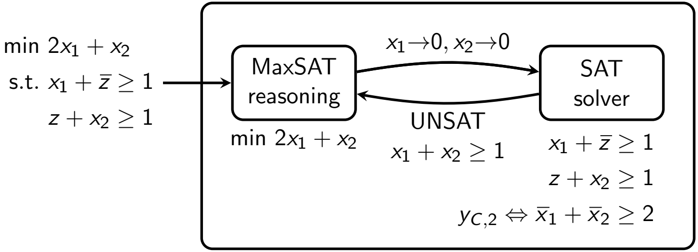

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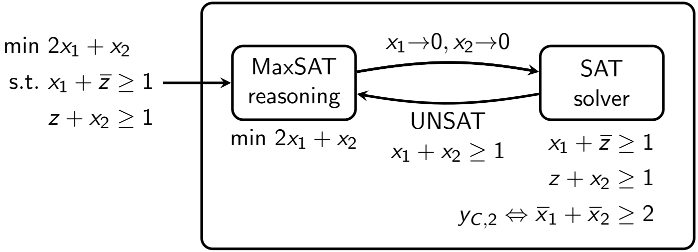

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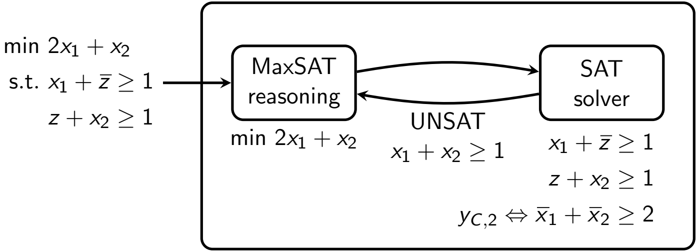
- Introduce counter variables $y_{C,1}$, fixed to 1, and $y_{C,2} \Leftrightarrow x_1 + x_2 \geq 2$
- Definition of counter variables encoded to CNF using totalizers
- Provide proof logging for totalizers leveraging [GMNO22, VDB22]

Example: Proof Logging for Core-Guided MaxSAT Solving (2/3)



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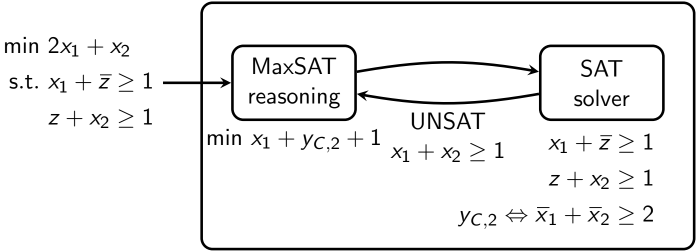

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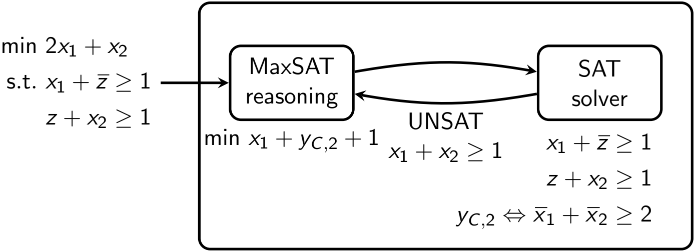

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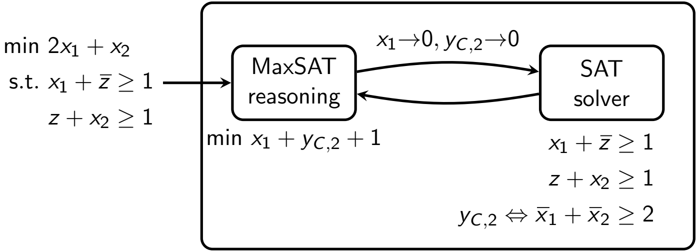

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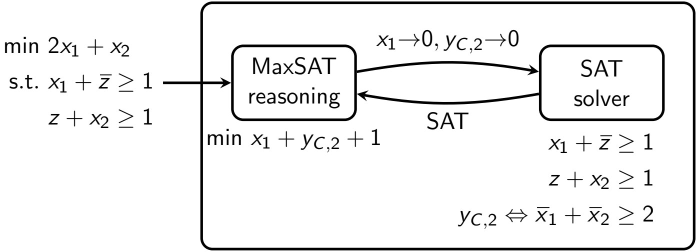

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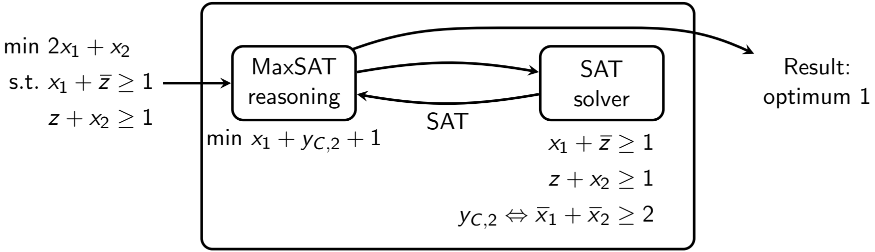
- Call SAT solver with assumptions $x_1 \to 0, y_{C,2} \to 0$
- SAT solver returns SAT with solution $x_1 \to 0, x_2 \to 1, y_{C,2} \to 0, z \to 0$
- Solution is logged in proof

Example: Proof Logging for Core-Guided MaxSAT Solving (3/3)



- Optimal solution found with value 1 ($x_1 \to 0, x_2 \to 1, y_{C,2} \to 0, z \to 0$)
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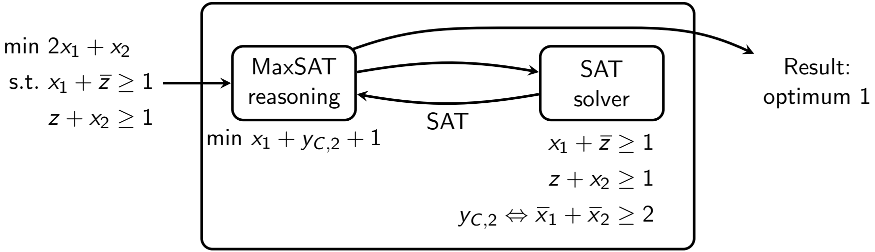

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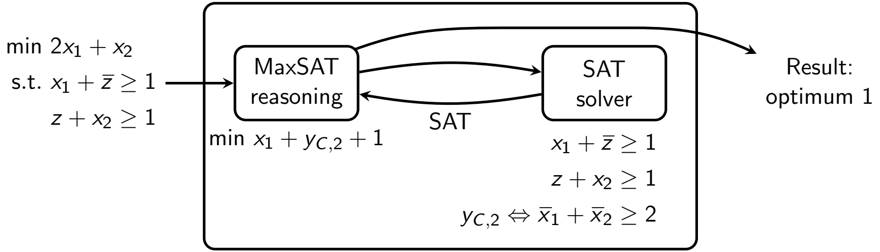

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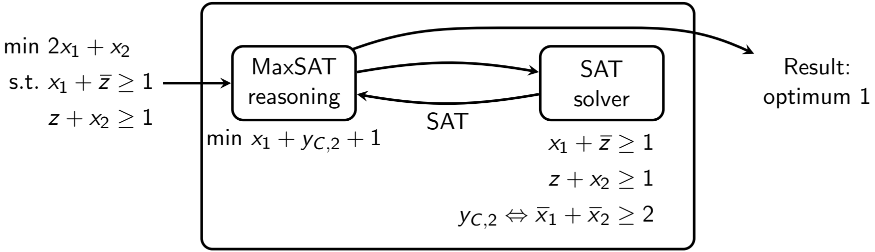

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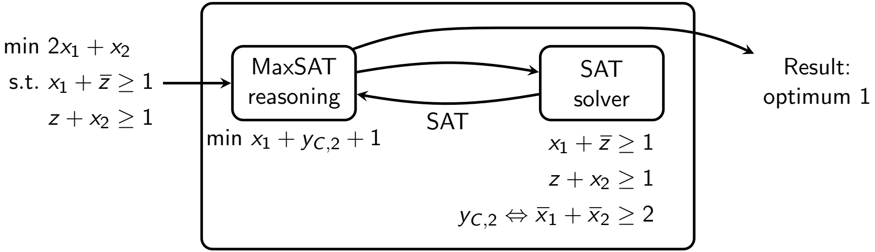

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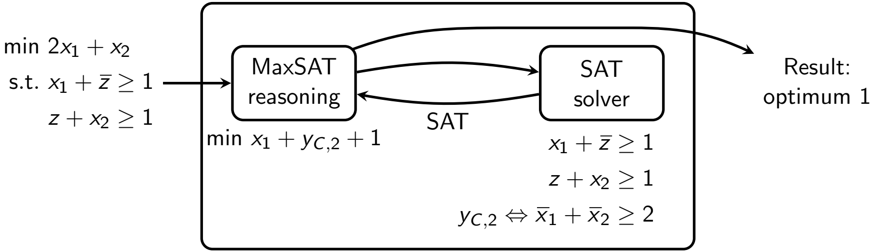

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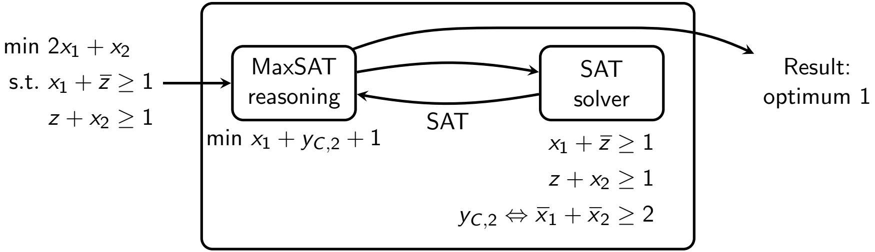
- Add invariant $x_1 + x_2 + \bar{y}_{C,2} \geq 2$ to objective-improving constraint $2\bar{x}_1 + \bar{x}_2 \geq 3$
- Result: $\bar{x}_1 + \bar{y}_{C,2} \geq 3$

Example: Proof Logging for Core-Guided MaxSAT Solving (3/3)



- Optimal solution found with value 1 ($x_1 \to 0, x_2 \to 1, y_{C,2} \to 0, z \to 0$)
- Prove optimality by assuming better solution exists, i.e., $2x_1 + x_2 \leq 0$
- Goal: Derive contradiction from this assumption
- Add invariant $x_1 + x_2 + \bar{y}_{C,2} \geq 2$ to objective-improving constraint $2\bar{x}_1 + \bar{x}_2 \geq 3$
- Result: $\bar{x}_1 + \bar{y}_{C,2} \geq 3$
- Contradicts assumption of solution with objective value < 1

General Approach: Proof Logging for Core-Guided MaxSAT Solving

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3. Obtain proof of optimality if $LB = UB$

Advanced Techniques for Core-Guided MaxSAT

- Additional techniques that are skipped in this talk
  - Intrinsic at-most-one constraints [IMM19]
  - Hardening [ABGL12]
  - Lazy counter variables [MJML14]
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- Proof logging also required for these techniques
- Very convenient to do in our proof format $\rightarrow$ see our paper [BBN$^+$23]

Experiments

- Implemented certifying version of state-of-the-art solver CGSS[1] [IBJ21]
- Proof checked with proof checker VeriPB[2]
- Benchmarks from MaxSAT Evaluation 2022[3]
  - 607 unweighted instances and 594 weighted instances

[1] https://gitlab.com/MIAOresearch/software/certified-cgss
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First result:

- Discovered bugs in CGSS (and also RC2, on which CGSS is based)
  - All claimed optimal solutions correct for our benchmarks set
  - But solver reasoning sometimes wrong
  - Solver bug could lead to erroneous claims of optimality for other benchmarks

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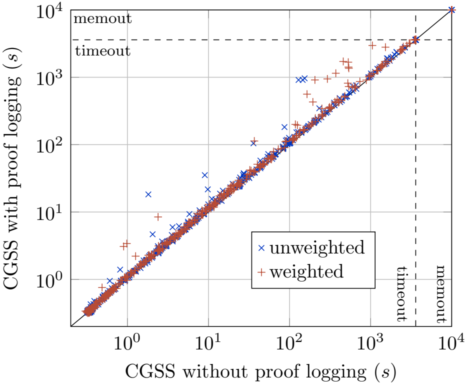
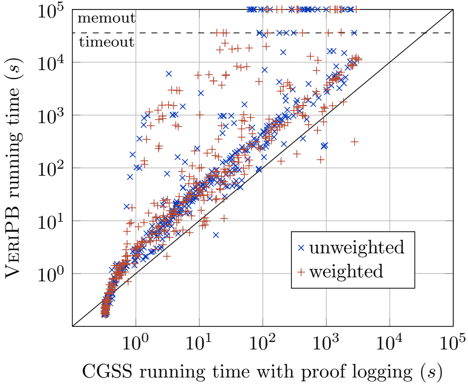
Experimental results



(a) Overhead for proof logging.



(b) Solving versus checking.

- Low proof logging overhead (8.8% median)
- Checking time could be improved (VeriPB not optimized for SAT solver proofs)

The Sales Pitch For Proof Logging

1. Certifies correctness of computed results
2. Detects errors even if due to compiler bugs, hardware failures, or cosmic rays
3. Debugging support during development [EG21, GMM$^+$20, KM21, BBN$^+$23]
4. Facilitates performance analysis
5. Helps identify potential for further improvements
6. Enables auditability
7. Serves as stepping stone towards explainability

Future Work

Further proof logging:

- State-of-the-art linear SAT-UNSAT search solver (like Pacose)
- Implicit hitting set MaxSAT solver
  - Fundamental challenge: proof logging for MIP solver
- Pseudo-Boolean optimization
- Other paradigms: constraint programming, MIP, planning, dynamic programming

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Improving performance and reliability:

- Optimize VeriPB for SAT solver proofs
- Backwards checking/trimming for verification (as in DRAT-trim [HHW13a])
- Formally verified proof checker [BMM+23]

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Sounds interesting? Join us! We are hiring!

Conclusion

- MaxSAT: successful optimization paradigm, but without proof logging
- PB reasoning supports MaxSAT proof logging
- This work: Proof logging for state-of-the-art core-guided MaxSAT solving
- Hopefully step towards general adoption of proof logging for MaxSAT

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Pseudo-Boolean reasoning provides unified proof logging method for:

- SAT solving (including advanced techniques) [GN21, BGMN22]
- Constraint programming [EGMN20, GMN22]
- Subgraph problems [GMN20, GMM+20]
- SAT-based pseudo-Boolean solving [GMNO22]
- This work: Core-guided MaxSAT solving

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Thank you for your attention!

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